REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
04-08-2011	Journal Article	

4. TITLE AND SUBTITLE

Analysis of Air Breathing Hall-Effect Thruster (Preprint)

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Leonid Pekker and Michael Keidar

5d. PROJECT NUMBER

5f. WORK UNIT NUMBER

50260542

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

ERC, Inc.
Air Force Research Laboratory (AFMC)
AFRL/RZSA
10 E. Saturn Blvd.
Edwards AFB CA 93524-7680

8. PERFORMING ORGANIZATION REPORT NUMBER

AFRL-RZ-ED-JA-2011-335

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory (AFMC)
AFRL/RZS
5 Pollux Drive
Edwards AFB CA 93524-7048

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S NUMBER(S)

AFRL-RZ-ED-JA-2011-335

12. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution unlimited (PA #11615).

13. SUPPLEMENTARY NOTES

For Publication in the Journal of Propulsion and Power

14. ABSTRACT

The principle idea of using air breathing electrical propulsion for a vehicle flying at orbital speed on the edge of Earth's atmosphere is examined. In this paper, we present a simple model of a Hall Effect thruster in which the propellant is an ambient air. The required lengths of the thruster chamber, the magnetic fields, the thrust, and other parameters of an ideal air breathing Hall Effect thruster are calculated as a function of the flying altitude of the vehicle. We demonstrated that an air breathing Hall thruster is indeed capable of providing the needed thrust for near space satellites in the case of the altitudes of about 80-90 km.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					Marcus P. Young
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER *(include area code)*
Unclassified	**Unclassified**	**Unclassified**	SAR	18	N/A

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. 239.18

STAFF SUMMARY SHEET

	TO	ACTION	SIGNATURE (Surname), GRADE AND DATE		TO	ACTION	SIGNATURE (Surname), GRADE AND DATE
1	STINFO	Log in SS/ DT/Copy	Technical Editor Aug 4, 2011	6	Lt Col Davis	Review/ Approve	*Signed & Dated Approval Page AFRL Chief Technical Advisor
2	Lt Col Davis	Review/ Sign	*Signed & Dated Form 79 AFRL Chief Technical Advisor	7	STINFO	Update SS	K. Gfeller 18 Aug 11 Technical Editor
3	STINFO	Approval Page / SS	Technical Editor 8 Aug 11	8	D. Spotts	Review for PAO	Deborah A Spotts STINFO Officer 18 Aug 11
4	Darrell Rhoden	Review/ Approve	Darrell Rhoden OPSEC/FDO 16 Aug 11	9	J. Haire/M. Mingledorf	Review/ Clear	Public Affairs Office
5	STINFO	Attach Rev Update SS	K. Gfeller 18 Aug 11 Technical Editor	10	STINFO	Notify/ Distribute	

SURNAME OF ACTION OFFICER AND GRADE	SYMBOL	PHONE	TYPIST'S INITIALS	SUSPENSE DATE
Dawn Hammel	AFRL/RZOI	5-5015		20110825

SUBJECT	DATE
AFRL-RZ-ED-JA-2011-335 (Distribution A)	20110804

SUMMARY

STINFO DISTRIBUTION TEAM:

1. Please remember the spreadsheet (mark the date you took this document to each person on this route)
2. Please - NO PAPERCLIPS!

ALL:

1. Please review attached document
2. Mark any comments and sign your signature block on the attached "Authorization for Release"
3. Place document in your "out box" for pick-up and routing by a member of the Distribution Team

APPROVAL PAGE

MEMORANDUM FOR RZS (In-House Publication)

FROM: RZOI (STINFO) (for RZSA) 4-Aug-11

SUBJECT: Authorization for Release of Technical Information, Control Number: **AFRL-RZ-ED-JA-2011-335**
Leonid Pekker (AFRL/ERC) and Michael Keidar (The George Washington University), "Analysis of Air Breathing Hall-Effect Thruster"

For publication in the *Journal of Propulsion and Power* **(Statement A)**
(Deadline: N/A)

1. This request has been reviewed by the Foreign Disclosure Office for: a.) appropriateness of distribution statement, b.) military/national critical technology, c.) export controls or distribution restrictions, d.) appropriateness for release to a foreign nation, and e.) technical sensitivity and/or economic sensitivity.
Comments: OK

Signature Darrell Rhoden DARRELL W. RHODEN Foreign Disclosure Officer Date 16 Aug 11

2. This request has been reviewed by the Public Affairs Office for: a.) appropriateness for public release and/or b) possible higher headquarters review.
Comments:

Signature Date

3. This request has been reviewed by the STINFO for: a.) changes if approved as amended, b) appropriateness of references, if applicable; and c.) format and completion of meeting clearance form if required
Comments: OK

Signature Date 4 Aug 11

4. This request has been reviewed by RZ for: a.) technical accuracy, b.) appropriateness for audience, c.) appropriateness of distribution statement, d.) technical sensitivity and economic sensitivity, e.) military/ national critical technology, and f.) data rights and patentability
Comments:

APPROVED / APPROVED AS AMENDED / DISAPPROVED

for LTC Davis 8 Aug 2011

LaDONNA J. DAVIS, Lt Col, USAF Date
Deputy Chief
Space and Missile Propulsion Division

TECHNICAL EDITING AND PUBLISHING REQUEST

(See Page 3 for Instructions)

TRACKING NO: AFRL-RZ-ED-JA-2011-335	DATE SENT TO DTIC:	PUBLICATION NO:

1. TYPE OF EDIT REQUESTED
[ ] N/A [ ] COMPLIANCE EDIT/NO PRINT [ ] COMPLIANCE EDIT/PRINT [ ] Edit: ([ ] Lite [ ] Full [ ] E-edit)

2a. CONTRACT NUMBER OR IN-HOUSE	2b. PROGRAM ELEMENT NO.	2c. PROJECT NO.	2d. TASK NO.	2e. WORK UNIT (JON) *Required*
In-house				50260542

3. WORK UNIT MANAGER	4. OFFICE SYMBOL	5. PHONE NO.	6a. REPORT DATE (MM/YY)
Marcus Young	RZSA	5-5624	08/11

7. CONTRACTOR/CONTRIBUTOR (if delivery order, list both prime and sub)
Leonid Pekker

6b. PERIOD COVERED (MM/DD/YY) *(for Tech Reports Only)*
FROM:
TO:

8. AUTHOR(S) (continue on next page, Block 22 (Remarks) if necessary)
Leonid Pekker and Michael Keidar

9. TITLE (include volume no., if applicable)
Analysis of Air Breathing Hall Effect Thrusters

10. SUBTITLE

11. PAGE COUNT *(for Tech Reports only)*

12a. PUB STATUS
For TRs: [ ] Special [ ] Interim [ ] Final (Closes JON) [ ] Other
For Journal Articles: [ ] Preprint [ ] Postprint

12b. PUB TYPE
[ ] Tech Report (TR) [X] Journal Article (JA) [ ] Abstract (AB) [ ] Tech Paper (TP) [ ] Viewgraphs (VG) [ ] Conference Paper (TP) [ ] Tech Note (TN) [ ] Tech Memo (TM) [ ] Multimedia (MM) [ ] Poster Session (PS) [ ] Other

12c. PUB INFORMATION (for Journal Articles, Conference Proceedings/Abstracts)
(May be completed in Field 22, next page)
If **Preprint**, to be submitted to:
If **Postprint**, to be submitted to:

13. DIGITAL PROVIDED? [X] Yes [ ] No

14. CLASSIFICATION [X] Unclassified [ ] Classified - Confidential [ ] Classified - Secret

15. WARNING NOTICES (for Classified Publications)
[ ] Restricted Data (RD) [ ] Not releasable to Foreign Nationals (NOFORN) [ ] Formerly Restricted Data (FRD) [ ] Other (Add to Remarks, Block 22)

16a. CLASSIFIED BY OR DERIVED FROM:

16B. DECLASSIFY ON:

Y	N	17. CONTENT RELATED INFORMATION (Check a response for each item)
[X]	[ ]	SF 298 Included *(STINFO Provides)*
[ ]	[X]	Notice (Signature Page) Included *(not required for TPs)*
[ ]	[X]	**Contains Export-Controlled Information**
[ ]	[X]	Contains Software Code (that can run a program)
[ ]	[X]	Contains Patentable Information (patent being pursued but not approved)

Y	N	18. APPLICABLE DATA RIGHTS (Check a response for each item)
[X]	[ ]	Unlimited Rights Information
[ ]	[X]	Limited Rights Information
[ ]	[X]	Government Purpose Rights Information
[ ]	[X]	SBIR Phase I [ ] II [ ] III [ ] Data Rights (check one)
[ ]	[X]	STTR Phase I [ ] II [ ] III [ ] Data Rights (check one)
[ ]	[X]	Copyright? If yes, provide year/owner:

19a. DISTRIBUTION STATEMENTS *(Check One)*

[X] A: Approved for public Release; distribution unlimited. PA Case No: Date Cleared:

[ ] B: Distribution authorized to U.S. Government agencies only; (reason); (date of determination). Refer other requests for this document to (controlling or originating DoD office).

[ ] C: Distribution authorized to U.S. Government agencies and their contractors;; (reason); (date of determination). Refer other requests for this document to controlling or originating DoD office).

[ ] D: Distribution authorized to Department of Defense and U.S. DoD contractors;; (reason); (date of determination). Refer other requests for this document to controlling or origiinating DoD office).

[ ] E: Distribution authorized to DoD components only; (reason); (date of determination). Refer other requests for this document to (controlling or origiinating DoD office).

[ ] F: Further dissemination only as directed by (controlling DoD office) (date of determination) or higher DoD authority.

[ ] X: Distribution authorized to U.S. Government Agencies and private individuals or enterprises eligible to obtain export-controlled Technical data in accordance with DoD Directive 5230.25 (date of determination). Controlling DoD office is (insert) AFRL/RZS

19b. REASON(S)
(Choose one for all distribution statements but Dist A)
(May check up to 3)

[ ] Administrative or Operational Use *(B,C,D,E)*
[ ] Contractor Performance Evaluation *(B,E)*
[ ] Critical Technology (Export-Control) *(B,C,D,E)*
[ ] Direct Military Support *(E)*
[ ] Foreign Government Information *(B,C,D,E)*
[ ] Premature Dissemination *(B, E)*
[ ] Proprietary Information *(B, E)*
[ ] Software Documentation *(B,C,D,E)*
[ ] Specific Authority *(B,C,D,E)*
[ ] Test and Evaluation *(B, E)*

20. WORK UNIT MANAGER'S SIGNATURE: *(for TPs submit with paper; for TRs sign during CRC approval process)*

21. BRANCH MANAGEMENT SIGNATURE: *(for TPs submit with paper; for TRs sign during CRC approval process)*

22. REMARKS (Provide any additional information)

This material/paper has been presented at 42nd AIAA Plasmadynamics and Laser Conference, Sheraton Waikiki Conference Center, Honolulu, Hawaii, June 27-30, 2011 and cleared for public release: clearance number is 11218 and STINFO tracking number is AFRL-RZ-ED-VG-2011-244 (viewgraphs) and clearance number is 11127 and STINFO tracking number is AFRL-RZ-ED-TP-2011-189 (conference paper).

If Conference Presentation:

Name of Conference: ________

Location of Conference: ________

Submission Deadline: ________

Conference Dates: ________

Printing Info (for Reports): No. Copies: ____ Paper (Color? ☐ Yes ☐ No) **No. CDs:** ____

GPC Name: ________ Phone: ________

DARPA ORDER NO. **N/A**

HUMAN USE PROTOCOL NO. **N/A**

ANIMAL USE PROTOCOL NO. **N/A**

23. Coordination (As Required) (At AFRL-West, submit with Journal Article or Conference Paper) (for TRs, wait until CRC Report is circulated for Signatures):

DATE	APPROVED / DISAPPROVED	TYPE / PRINT NAME	SIGNATURE
Click here to enter a date.	☐ APPROVED ☐ DISAPPROVED		
Click here to enter a date.	☐ APPROVED ☐ DISAPPROVED		
Click here to enter a date.	☐ APPROVED ☐ DISAPPROVED		
Click here to enter a date.	☐ APPROVED ☐ DISAPPROVED		
Click here to enter a date.	☐ APPROVED ☐ DISAPPROVED	Branch Technical Reviewer (If Different from PM) - *Optional*	
Click here to enter a date.	☐ APPROVED ☐ DISAPPROVED	Technical Reviewer's Branch Chief (If Different from PM) - *Optional*	
Click here to enter a date.	☒ APPROVED ☐ DISAPPROVED	RZS Technical Advisor	[signature] for LTC Davis

Submission Deadline *(required)*:

Are any other agencies or sponsors also reviewing this document?

(•) No ( ) Yes If "Yes" please list name(s) / Company

If Article will be Published:

Name of Publication:	Journal of Propulsion and Power
Estimated Publication Date:	Summer 2012

TO BE COMPLETED BY ON-SITE CONTRACTOR (If any On-Site Contractors are Listed as Author):
(Applicable only to on-site contractors without H029 contract clause;
a Request-to-Publish Letter is required from all other contractors)

☒ The material in this document is technically accurate

☒ The material in this document contains no Proprietary information

☒ The material in this document contains no Classified information

☒ If Distribution A is selected on Page 1 of this form, we request Public Release of this document

Site Manager or Contractor Subject Matter Expert (not Author) Signature: 2 AUG 2011

Analysis of Air Breathing Hall-Effect Thruster

Leonid Pekker[1]
ERC Inc., Edwards AFB, CA 93524

and

Michael Keidar[2]
The George Washington University, Northwest Washington, DC 20052

The principle idea of using air breathing electrical propulsion for a vehicle flying at orbital speed on the edge of Earth's atmosphere is examined. In this paper, we present a simple model of a Hall Effect thruster in which the propellant is an ambient air. The required lengths of the thruster chamber, the magnetic fields, the thrust, and other parameters of an ideal air breathing Hall Effect thruster are calculated as a function of the flying altitude of the vehicle. We demonstrated that an air breathing Hall thruster is indeed capable of providing the needed thrust for near space satellites in the case of the altitudes of about 80 – 90 km.

Nomenclature

A	=	mass of a heavy particle, in au
B	=	strength of the magnetic field
D_{diff}	=	ambient gas diffusion coefficient
E	=	strength of the electric field
E_{jet}	=	energy flow leaving the Hall thruster with the plasma jet
E_{anode}^{loss}	=	heat losses at the anode
E_{wall}^{loss}	=	energy losses at the wall
E_{ioniz}^{cost}	=	ionization cost
e	=	electron charge
F_{drag}	=	drag force
f_{drag}	=	drag factor
j_e	=	electron current density
j_i	=	ion current density
H	=	altitude of satellite orbit
L	=	length of the Hall thruster chamber
$\ln\Lambda$	=	Coulomb logarithm
M	=	mass of a heavy particle
M_i	=	ion mass
m_e	=	mass of an electron
n_e	=	electron number density
n_{gas}	=	number density of ambient gas

---

[1] Senior Research Staff Engineer, leonid.pekker.ctr@edwards.af.mil.
[2] Associate Professor, keidar@gwu.edu.
Distribution A: Approved for public release; distribution is unlimited

n_i	=	ion number density
n_{pl}	=	number density of plasma
P	=	electric power put into the discharge
r_a	=	inner radius of the opening of the air breathing Hall thruster
r_b	=	outer radius of the opening of the air breathing Hall thruster
r_D	=	Debye radius
r_L	=	electron cyclotron radius
S	=	effective cross-section of the satellite due to drag force
T_e	=	electron temperature
$T_{e,jet}$	=	electron temperature in the plasma jet
T_{gas}	=	ambient gas temperature
$T_{i,jet}$	=	ion temperature in the plasma jet
$Thrust$	=	thrust
V_B	=	Bohm velocity
$V_{e,drift}$	=	electron drift velocity
V_{Te}	=	electron thermal velocity
V_{Tgas}	=	thermal velocity of ambient gas
V_0	=	the first cosmic velocity, 7.8 km/s
γ	=	coefficient of secondary electron emission
ΔV	=	delta-V (change in the velocity)
λ_{gas}	=	ambient gas mean free path
μ_e	=	electron mobility
σ_{gas}	=	collision cross-section of ambient gas molecules
σ_{ioniz}	=	ionization cross-section
τ_{e-life}	=	electron lifetime in Hall thruster
τ_{ioniz}	=	ionization time
τ_{wall}	=	characteristic time for gas molecule collisions with wall
$\tau_0 = L/V_0$	=	time that a gas molecule is flying through the thruster chamber
ν_{ei}	=	electron-ion collision frequency
ϕ_0	=	the Bohm sheath potential
φ	=	applied voltage
ω_{He}	=	electron cyclotron frequency

I. Introduction

This work is motivated by increasing interest in military and civil spacecrafts flying at the altitudes of 70 - 120 km. Since the drag at such altitudes is still significant, the thrusters of such a satellite should work continuously in order to maintain the orbit altitude. This demands that a significant amount of propellant be stored on-board satellites that use ordinary thrusters for propulsion. However, for such spacecrafts, using air breathing thrusters, in which the propellant is ambient air, looks very attractive, particularly for long-duration missions; air breathing thrusters allow significantly increased payload-to-weight ratio for such satellites. Using air breathing thrusters in Mars and other planets atmosphere becomes even more attractive because of multi-years satellite missions and high cost of payloads.

In a recent paper [1], Diamant has proposed a two-stage cylindrical Hall-Effect thruster for air breathing electric propulsion, in which the first stage is an electron cyclotron resonance ionization stage and the second stage is a cylindrical Hall thruster. The author built such a thruster and demonstrated its operation in xenon gas. As follows from his assumptions, his two-stage cylindrical Hall thruster is able to work at a 220 km orbit with ambient air passively compressed by a factor of 500 [1]. However, achieving such a large passive gas compression seems to be difficult and will be associated with substantial drag. In the BUSEC conception of an air breathing Hall thruster [2], the compression of air is achieved by a diffuser. It is not clear what compression ratio of the ambient air can be achieved by using such a diffuser at orbits of about 100 km, where the gas mean free paths are tens of centimeters and comparable with the dimensions of the thruster. Furthermore, the use of a diffuser can significantly increase the drag because gas is reflected diffusely in space [3].

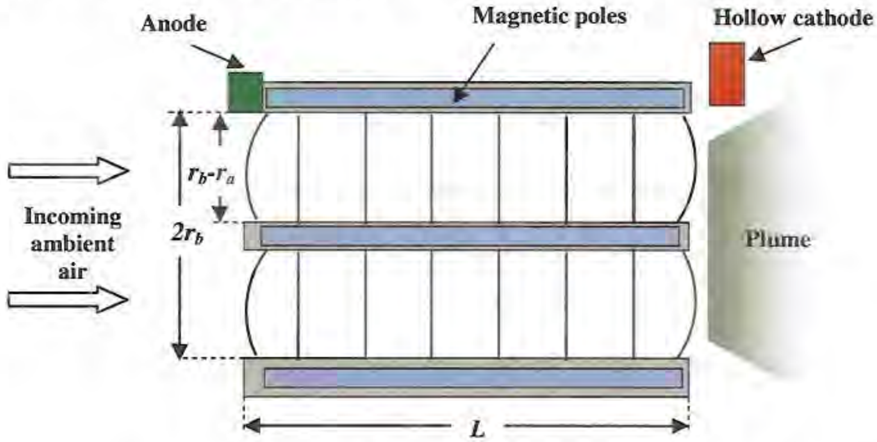

Figure 1. Schematic of air breathing Hall thruster (not to scale).

In the present work we propose a simple configuration of an air breathing Hall-Effect thruster, (shown in Figure 1), in which the incoming air flow is ionized and accelerated directly in the Hall thruster chamber without preliminary compression as in [1,2]. Such a design leads to a decrease of the possible drag, however its feasibility warrants careful consideration, which is the subject of this paper. The description of the model and numerical results are provided in Sections II and III, respectively, and the conclusions are given in Section IV.

II. Description of the Model

The following assumptions are made in the model: (1) no drag due to ambient gas passing through the Hall thruster chamber, illustrated in Figure 1, corresponding to a condition in which the ambient gas freely flows through the thruster chamber without a significant interactions with the wall; (2) the gas jet leaving the thruster chamber is fully ionized providing the maximum thrust; (3) the plasma in the Hall thruster is quasineutral, $n_i = n_e = n_{pl}$, corresponding to a condition in which r_D is smaller than the characteristic dimensions of the thruster; in our case, Figure 1, r_D must be smaller than $r_b - r_a$; (4) the electron cyclotron radius is much smaller than the Hall thruster gap, $r_L < r_b - r_a$, as is required for confinement of electrons in the Hall thruster.

The first assumption is well satisfied when the characteristic time required for a gas molecule to reach the wall and transfer its momentum, τ_{wall}, is larger than the time required for the molecule to freely fly through the thruster chamber, $\tau_0 = L / V_0$, i.e.

$$\frac{\tau_{wall} \cdot V_0}{L} > 1 \ , \tag{1}$$

In the case of free molecular flow, when the gas mean free path is larger than $r_b - r_a$, Figure 1, τ_{wall} can be estimated as

$$\tau_{wall} = \frac{r_b - r_a}{V_{Tgas}} \ , \tag{2}$$

and in the case of collisional molecular flow, $r_b - r_a < \lambda_{gas}$, as

$$\tau_{wall} = \frac{(r_b - r_a)^2}{D_{diff}} = \frac{(r_b - r_a)^2}{\lambda_{gas} \cdot V_{Tgas}} = \frac{n_{gas} \cdot \sigma_{gas} \cdot (r_b - r_a)^2}{V_{Tgas}} . \tag{3}$$

Here, we have substituted $(n_{gas} \cdot \sigma_{gas})^{-1}$ for λ_{gas}. Combining Eqs. (2) and (3) we obtain

$$\tau_{wall} = Max\left[\frac{r_b - r_a}{V_{Tgas}}, \frac{n_{gas} \cdot \sigma_{gas} \cdot (r_b - r_a)^2}{V_{Tgas}}\right] . \tag{4}$$

The second assumption is fulfilled when the ionization time of a gas molecule,

$$\tau_{ioniz} = \frac{1}{V_{Te} \cdot n_e \cdot \sigma_{ioniz}} , \tag{5}$$

is much smaller than τ_0. Let us estimate τ_{ioniz}. The equation describing the mass conservation law for the heavy particles can be written as

$$V_0 \cdot n_{gas} = n_{pl} \cdot \sqrt{V_0^2 + \frac{2 \cdot e \cdot \varphi}{M_i}} , \tag{6}$$

where the left-hand side in Eq. (6) is the flux of incoming gas and the right-hand side is the ion flux leaving the chamber. In Eq. (6) we have assumed that the downstream plasma in the thruster chamber is quasineutral and fully ionized.

It should be stressed that in the model we assume the ambient gas is pure molecular nitrogen. This is a reasonable assumption since the air contains 78 percent of nitrogen and 21 percent of oxygen and their electron impact ionization cross sections are similar [4]. In the model, we neglect the dissociation of molecular nitrogen assuming that $M = M_i$. Since for electron energies between 20 – 80 eV the ionization and dissociation cross sections of molecular nitrogen are close and the ionization cross section of molecular nitrogen is much larger than the ionization cross section of atomic nitrogen [5], neglecting of the dissociation of nitrogen should not lead to a significant error and is reasonable for our simple model.

Since $e \cdot \varphi$ is usually much larger than the kinetic energy of an ambient gas molecule at the entrance plane of the Hall thruster (for nitrogen molecules it is about 8.75 eV), the n_{pl} can be written as

$$n_{pl} = n_e = \frac{V_0 \cdot n_{gas}}{\sqrt{\frac{2 \cdot e \cdot \varphi}{M}}} . \tag{7}$$

Substituting Eq. (7) and V_0 as 7.8 km/s into an Eq. (5), we obtain that condition $\tau_0 >> \tau_{ioniz}$ is fulfilled when

$$47.6 \cdot L \cdot n_{gas} \cdot \sigma_{ioniz}(T_e) \cdot \left(\frac{\varphi}{A \cdot T_e}\right)^{1/2} > 1 , \tag{8}$$

where T_e in Eq. (8) is measured in eV.

On the other hand, the electron lifetime in the Hall thruster, τ_{e-life}, must be larger than the ionization time for an electron, $(V_{Te} \cdot n_{gas} \cdot \sigma_{ioniz})^{-1}$; otherwise, the electron will leave the thruster chamber before producing any ionization events; τ_{e-life} can be estimated as

$$\tau_{e-life} = \frac{L}{V_{e,drift}} = \frac{L}{\mu_e \cdot E} \cdot \left(1 + \frac{\omega_{He}^2}{\nu_{ei}^2}\right), \tag{9}$$

It worth noting that in Eq. (9) we have assumed the classical electron transport across the magnetic field, and neglected the collisions of electrons with neutral particles (since a fully ionized plasma is assumed); recall that in the case of a high magnetic field, electron transport in Hall thrusters can approach the classical limit [6]. Substituting in Eq. (9)

$$\mu_e = \frac{e}{m_e \cdot \nu_{ei}}, \quad \nu_{ei} = 2.1 \cdot 10^{-12} \cdot \ln\Lambda \cdot n_{pl} \cdot T_e^{-3/2}, \quad \omega_{He} = 1.76 \cdot 10^7 \cdot B, \quad E = \varphi / L, \tag{10}$$

and taking into account that ω_{He} / ν_{ei} in Hall thrusters is much larger than one, we obtain

$$\tau_{e-life} = 1.5 \cdot 10^{15} \cdot \frac{L^2 \cdot B^2}{\sqrt{\varphi \cdot A} \cdot \ln\Lambda \cdot n_{gas} \cdot T_e^{-3/2}}, \tag{11}$$

where B is measured in Gauss, and T_e in eV. Now, taking into account that τ_{e-life}, Eq. (11), must be larger than $(V_{Te} \cdot n_{gas} \cdot \sigma_{ioniz})^{-1}$ we obtain that the following condition on L must also be satisfied to fulfill assumption 2:

$$4 \cdot 10^{10} \cdot L \cdot B \cdot T_e \cdot \left(\frac{\sqrt{\varphi \cdot A} \cdot \ln\Lambda}{\sigma_{ioniz}(T_e)}\right)^{-1/2} > 1 \,. \tag{12}$$

Thus, we have shown that if the conditions given by Eqs. (8) and (12) are well satisfied, then the gas in the air breathing Hall thruster is well ionized.

Now let us examine the third assumption of the model: that the plasma is quasineutral. Substituting n_{pl} from Eq. (7) into an equation for the Debye radius [7],

$$r_D = 7.43 \cdot 10^3 \cdot \left(\frac{T_e}{n_{pl}}\right)^{1/2}, \tag{13}$$

and assuming that the gas is fully ionized, we obtain the following equation for the Debye radius:

$$r_D = 10^4 \cdot \left(\frac{T_e \cdot \sqrt{\varphi}}{n_{gas} \cdot \sqrt{A}}\right)^{1/2}. \tag{14}$$

Thus, if the Debye radius, Eq. (14), is much smaller than the characteristic dimension of the thruster,

$$r_b - r_a > 10^4 \cdot \left(\frac{T_e \cdot \sqrt{\varphi}}{n_{gas} \cdot \sqrt{A}}\right)^{1/2}, \tag{15}$$

the plasma in the Hall thruster can be considered quasineutral; T_e in Eqs. (13) – (15) is measured in electron volts.

Assumption 4 of the model requires that the characteristic electron cyclotron radius be much smaller than the thruster gap. Substituting in an expression for electron cyclotron radius V_{Te} for electron transfer velocity, we obtain

$$\frac{r_b - r_a}{r_L} = 42 \cdot \frac{(r_b - r_a) \cdot B}{T_e^{1/2}} > 1 \ , \tag{16}$$

where the electron temperature is in electron volts and the magnetic field is in Gauss.

Assuming that all assumptions of the model are fulfilled, the thrust of an air breathing Hall thruster can be estimated as

$$Thrust = \pi \cdot (r_b^2 - r_a^2) \cdot M \cdot V_0 \cdot n_{gas} \cdot \sqrt{\frac{2 \cdot e \cdot \varphi}{M}} \tag{17}$$

and the electric power supplied to the discharge as

$$P = \pi \cdot (r_b^2 - r_a^2) \cdot (j_e + j_i) \cdot \varphi = \pi \cdot (r_b^2 - r_a^2) \cdot \left(e \cdot n_{pl} \cdot V_{e,drift} + e \cdot n_{gas} \cdot V_0\right) \cdot \varphi \ , \tag{18}$$

where $\pi \cdot (r_b^2 - r_a^2)$ is the opening area of the thruster, and $M \cdot V_0 \cdot n_{gas}$ and $\Delta V = \sqrt{2 \cdot e \cdot \varphi / M}$ in Eq. (17) are correspondingly the mass flux entering the thruster and the increase in the velocity of heavy particles due to their acceleration in the Hall thruster chamber. Since the third assumption of the model is that the plasma is quasineutral, in Eq. (18) in the expression for j_e we have used n_{pl} instead of n_e. Substituting in Eq. (17) $V_{e,drift} = \mu_e \cdot E \cdot \nu_{ei}^2 / \omega_{He}^2$, n_{pl} from Eq. (7), and $V_0 = 7.8$ km/s and using Eqs. (10), we obtain that

$$j_e = 6.01 \cdot 10^{-35} \cdot \frac{\ln\Lambda \cdot T_e^{-3/2} \cdot A \cdot n_{gas}^2}{B^2 \cdot L} \ , \tag{19}$$

$$j_i = 1.25 \cdot 10^{-15} \cdot n_{gas} \ , \tag{20}$$

where T_e is in eV, B in Gauss.

Assuming diffuse reflection of gas molecules from the satellite walls [3], the drag force can be estimated as,

$$F_{drag} = M \cdot V_0^2 \cdot n_{gas} \cdot S \ . \tag{21}$$

It should be stressed that since the gas entering the thruster does not produce drag, the first assumption of the model, the opening area of the thruster, $\pi \cdot (r_b^2 - r_a^2)$, is not included in S.

One of the important parameters governing the discharge in a Hall thruster is the electron temperature. Now let us obtain an equation for electron temperature for an air breathing Hall thruster. An energy conservation equation in the Hall thruster chamber can be written as

$$\pi \cdot (r_b^2 - r_a^2) \cdot \left(e \cdot n_{pl} \cdot V_{e,drift} + e \cdot n_{gas} \cdot V_0\right) \cdot \varphi = E_{wall}^{loss} + E_{jet} + E_{anode}^{loss} \ , \tag{22}$$

where the left-hand side of Eq. (22) is the electric power supplied to the discharge, Eq. (18), while the right-hand of this equation describes the heat losses at the Hall thruster walls and at the anode, and the energy flow leaving the Hall thruster with the plasma jet. Assuming no secondary electron emission from the wall, $\gamma = 0$, this corresponds to the case of minimal heat loss to the wall; equations for E_{wall}^{loss}, E_{jet}, and E_{anode}^{loss} can be written as

$$E_{wall}^{loss} = 2\cdot\pi\cdot(r_a + r_b)\cdot L\cdot\left[n_{pl}\cdot V_B\cdot e\cdot\left(\phi_0 + E_{ion}^{\cos t}\right) + 2\cdot T_e\cdot n_{pl}\cdot\exp\left(-\frac{e\cdot\phi_0}{T_e}\right)\cdot\sqrt{\frac{T_e}{2\cdot\pi\cdot m_e}}\right], \quad (23)$$

$$E_{jet} = \pi\cdot(r_b^2 - r_a^2)\cdot V_0\cdot n_{gas}\cdot e\cdot\left(\varphi + T_{i,jet} + T_{e,jet} + E_{ioniz}^{\cos t}\right), \quad (24)$$

$$E_{anode}^{loss} = \pi\cdot(r_b^2 - r_a^2)\cdot n_{pl}\cdot V_{e,drift}\cdot\frac{3}{2}\cdot e\cdot T_e\,. \quad (25)$$

In Eq. (23) we have used the Bohm plasma sheath theory [8] and in Eq. (25) have taken into account the heat flux that the electrons bring to the anode, and neglected the (typically small) ion flux to the anode. The first term in the square brackets on the right-hand side of Eq. (23) describes the heat flux to the wall due to the recombination process plus the kinetic energy flux that ions bring to the wall, and the second term describes the heat flux that electrons bring to the wall. The plasma sheath potential drop, ϕ_0, and the Bohm velocity, V_B [8], are

$$\phi_0 = \frac{1}{2}\cdot T_e\cdot\ln\left(\frac{2\cdot\pi\cdot m_e}{M}\right), \quad (26)$$

$$V_B = \sqrt{\frac{T_e}{M}}\;. \quad (27)$$

It should be stressed that in our model we have actually divided ions into two parts: the fast ions in the core of the thruster chamber (these ions are accelerated by applied electric field and do not reach the wall) and the slow ions at the vicinity of the wall which reach the wall. The ions created at the wall are accelerated by the sheath potential and reach the wall, where they recombine with electrons and then move back into the chamber as neutrals, and then, they are again ionized in the vicinity of the wall and move back to the wall. Since in our model the neutral particle ionization length is assumed to be much smaller than L, these newborn ions do not gain much energy from the applied electrical field in this cycle. Therefore, we have neglected the kinetic energy of these ions at the sheath edge as it has been assumed in [8].

Since the ion and electron temperatures in the plasma jet (in the plume, Figure 1) are typically a few electron volts and much smaller than the ionization cost, which is 20 – 40 eV, we will further neglect $T_{i,jet}$ and $T_{e,jet}$ in Eq. (24). Substituting the reduced equations for E_{wall}^{loss}, E_{jet}, E_{anode}^{loss} into Eq. (22), after some algebra we obtain a final equation for the electron temperature:

$$\begin{aligned} &4.9\cdot10^{-20}\cdot\frac{A\cdot n_{gas}\cdot\ln\Lambda}{L^2\cdot B^2}\cdot\left(\frac{\varphi}{T_e}\right)^{3/2}\cdot\left(1-\frac{3\cdot T_e}{2\cdot\varphi}\right) = \\ &= \frac{E_{ioniz}^{\cos t}}{L}\cdot\sqrt{\varphi} + \frac{1}{(r_b - r_a)}\cdot 1.5\cdot T_e^{1/2}\cdot\left(E_{ioniz}^{\cos t} + \left[2+\ln\left(17\cdot A^{1/2}\right)\right]\cdot T_e\right) \end{aligned}. \quad (28)$$

Thus, knowing the parameters of the air breathing Hall thruster (L, r_a, r_b, B, φ) and parameters of the ambient air (A, $\sigma_{ioniz}(T_e)$, σ_{gas}, n_{gas}, T_{gas}, $E_{ioniz}^{\cos t}$) using Eq. (28) we can calculate T_e, then check the assumptions made in the model, Eqs. (1), (8), (12), (15) and (16), and finally calculate the thrust, Eq. (17), the power, Eq. (18), the ion and electron currents, Eqs. (19) and (20), and other parameters of the air breathing Hall Effect thruster at a given satellite orbit.

III. Numerical Results

In the numerical results presented in this section the collision cross section for molecular nitrogen gas, σ_{gas}, was taken as $4.4\cdot10^{-19}$ m^2 [9]; this corresponds to

$$\lambda_{gas} = \frac{1.63 \cdot 10^{-5} \cdot T[K]}{P[Pa]} . \tag{29}$$

The ionization cross section of molecular nitrogen as a function of electron temperature, Figure 2, was calculated using data [5], and $\ln \Lambda$ was taken as 13.5; the length and the inner and outer radii of the Hall thruster chamber were chosen as 0.5, 0.03, and 0.05 m respectively. The calculations have been performed for two applied voltages, 3 and 30 kV, and for satellite orbits of 80 and 90 km.

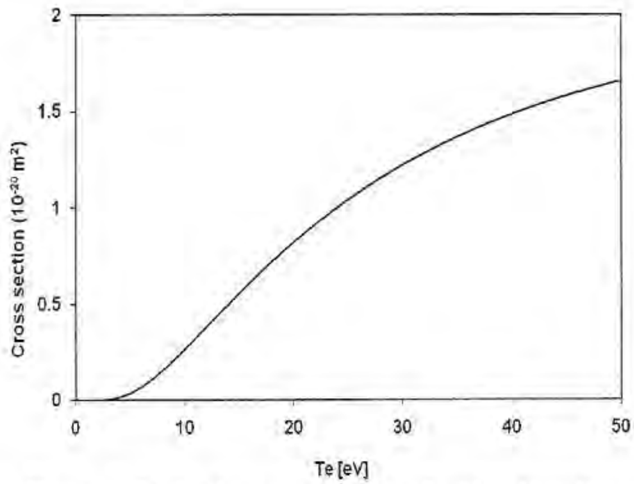

Figure 2. Ionization cross section of molecular nitrogen

Figure 3 shows the drag factor $f_{drag} = \tau_{wall} \cdot V_0 / L$ calculated for 80, 90, 100, and 110 km satellite orbits. As one can see, the first assumption of the model is well fulfilled only for the 80 km satellite orbit. However, since the thrust of air breathing Hall thrusters, Eq. (17), is generally larger than the maximal drag $\pi \cdot (r_b^2 - r_a^2) \cdot M \cdot n_{gas} \cdot V_0^2$ that might be produced by the air flowing through the Hall chamber, this assumption is not critical for the air breathing Hall thruster concept. It is worth noting that the gas flow through the thruster is free-molecular, Eq. (2), for $H = 90$ km and larger, and is collisional, Eq. (3), for $H = 80$ km and smaller. Obviously, for different thruster chamber length and width the transition between the free molecular and the collisional flows can take place at different orbits.

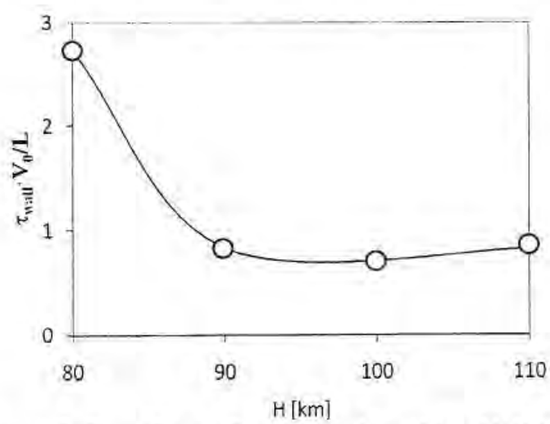

Figure 3. Drag factor for the air breathing Hall thruster: $L = 0.5$ m, $r_b = 0.05$ m, and $r_a = 0.03$ m

Figures 4 and 5 show the ratios of τ_0 / τ_{ioniz}, $\tau_{e-life} \cdot V_{Te} \cdot n_{gas} \cdot \sigma_{ioniz}$, and $(r_b - r_a)/r_L$, Eqs. (8), (12), and (16), calculated for the satellite orbits of 90 and 80 km respectively, the ionization cost of 40 eV, and the applied voltages of 3 kV and 30 kV; they correspond to the second and forth assumptions of the model. As one can see for the satellite orbit of 90 km these assumptions are satisfied for magnetic fields in the ranges of 10 – 30 and 20 – 70 Gauss correspondingly for $\varphi = 3$ kV and 30 kV; and for the orbit of 80 km in the ranges of 30 – 120 G for $\varphi = 3$ kV and for $\varphi = 30$ kV in all region of magnetic fields, 50 – 300 G. The third assumption of the model, Eq. (15), and $\omega_{He} / \nu_{ei} >> 1$ condition are very well satisfied in all ranges of magnetic field, potential, and orbit considered in the paper.

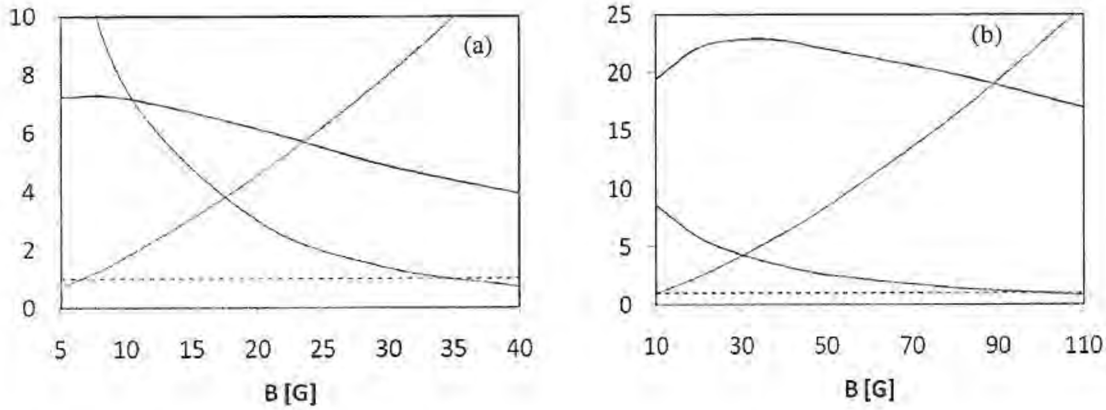

Figure 4. The τ_0 / τ_{ioniz} - blue line, $\tau_{e-life} \cdot V_{Te} \cdot n_{gas} \cdot \sigma_{ioniz}$ - green line, $(r_b - r_a)/r_L$ - red line calculated for $H = 90\,km$ and $\varphi = 3\,kV$ - (a) and $\varphi = 30\,kV$ - (b); the broken line correspond to unity.

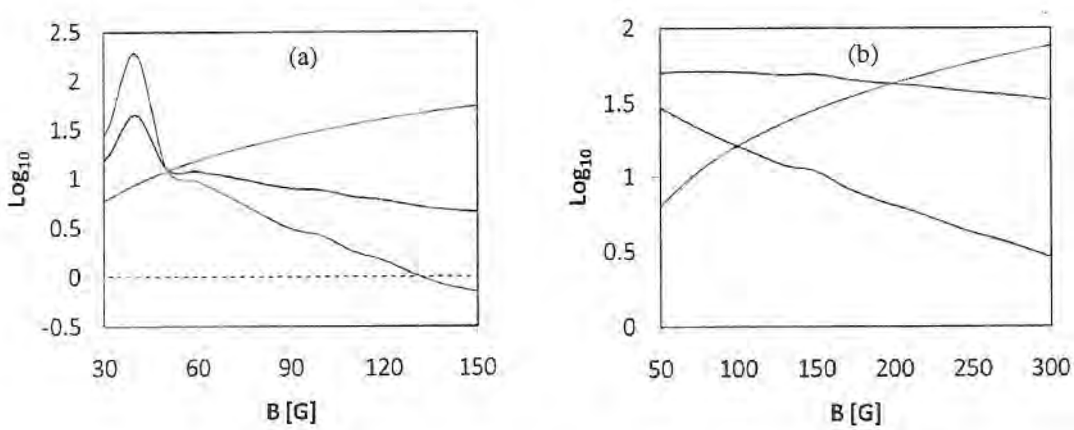

Figure 5. The τ_0 / τ_{ioniz} - blue line, $\tau_{e-life} \cdot V_{Te} \cdot n_{gas} \cdot \sigma_{ioniz}$ - green line, $(r_b - r_a)/r_L$ - red line calculated for $H = 80\,km$ and $\varphi = 3\,kV$ - (a) and $\varphi = 30\,kV$ - (b).

Since the air breathing Hall thruster plasma in our model is assumed to be fully ionized, the thrust, Eq. (20), is determined only by the ambient gas number density and the applied voltage, and the ion current, Eq. (17), is determined by the ambient gas number density only. The model gives $j_i = 8.9 \cdot 10^4$ and $4.8 \cdot 10^5$ A/m^2 for H = 90 and 80 km respectively; and *Thrust* = 18.63, 58.94, 100.5, and 317.9 N correspondingly for H = 90 km, φ = 3 and

30 kV and H = 80 km, φ = 3 and 30 kV. The obtained results are very understandable: with a decrease in the satellite orbit, the ambient gas density increases, leading to an increase in the ion current and the thrust; and an increase in the applied voltage leads to larger ΔV and, therefore, larger thrust. Of course, a lower orbit will lead to a larger drag force that the thrust will need to compensate for. The drag will depend on the satellite geometry; this paper considers the generic satellite, thus, specific drag estimations go beyond the scope of this paper.

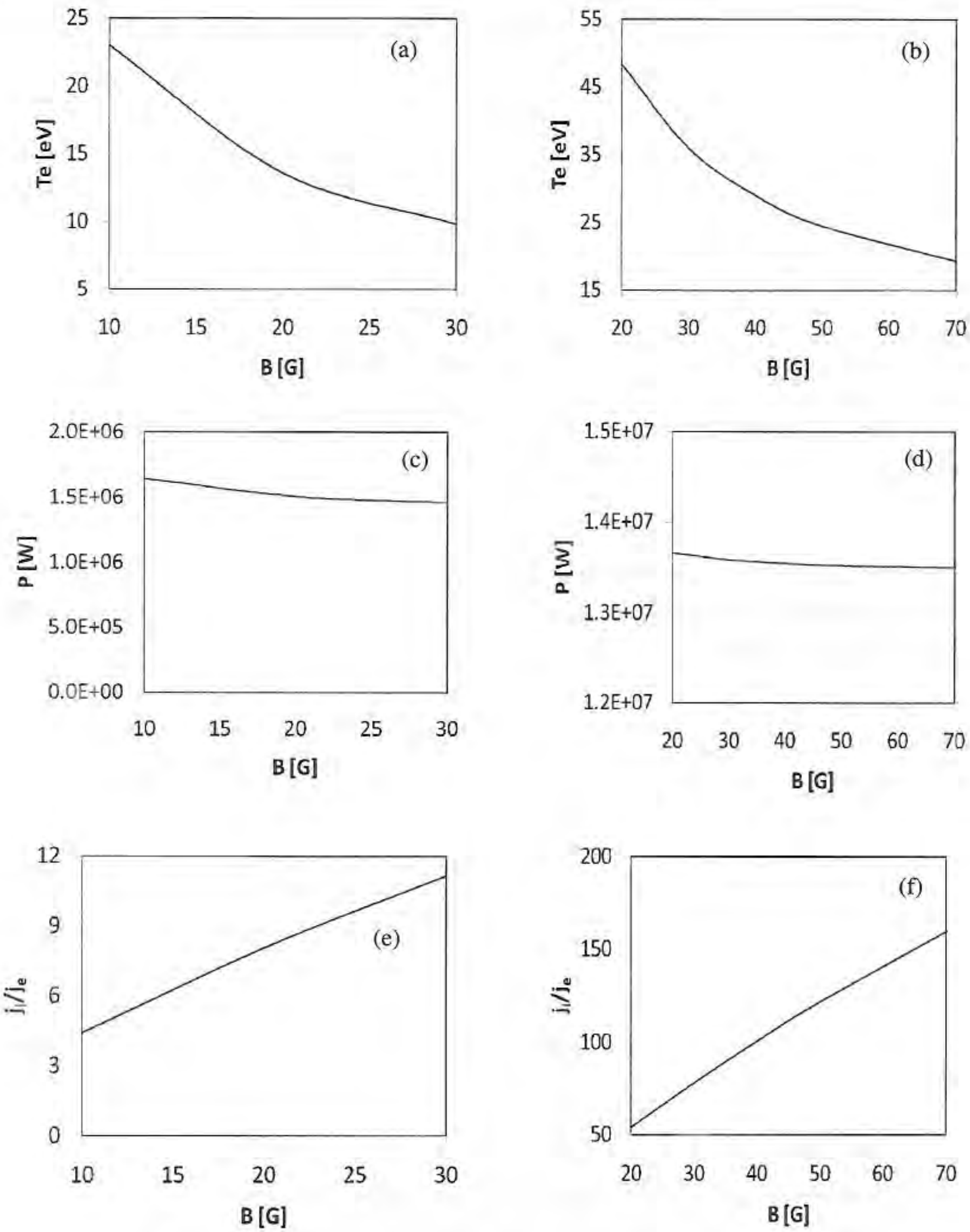

Figure 6. Parameters of the air breathing thruster for H = 90 km: plots (a), (c) and (e) correspond to $\varphi = 3\,kV$ and plots (b), (d) and (f) to $\varphi = 30\,kV$.

Figures 6 and 7 show the electron temperature, power, and j_i / j_e vs. magnetic field for $H = 90$ and 80 km respectively. In these figures we have selected magnetic fields for which the model assumptions are satisfied. As was expected with an increase in the applied voltage, the electron temperature increases, and with an increase in the magnetic field it decreases, as shown in Figures 6a, 6b and 7a and 7b. Since the ion current in the model is independent of the magnetic field (ions are not magnetized), while the electron current decreases with an increase in

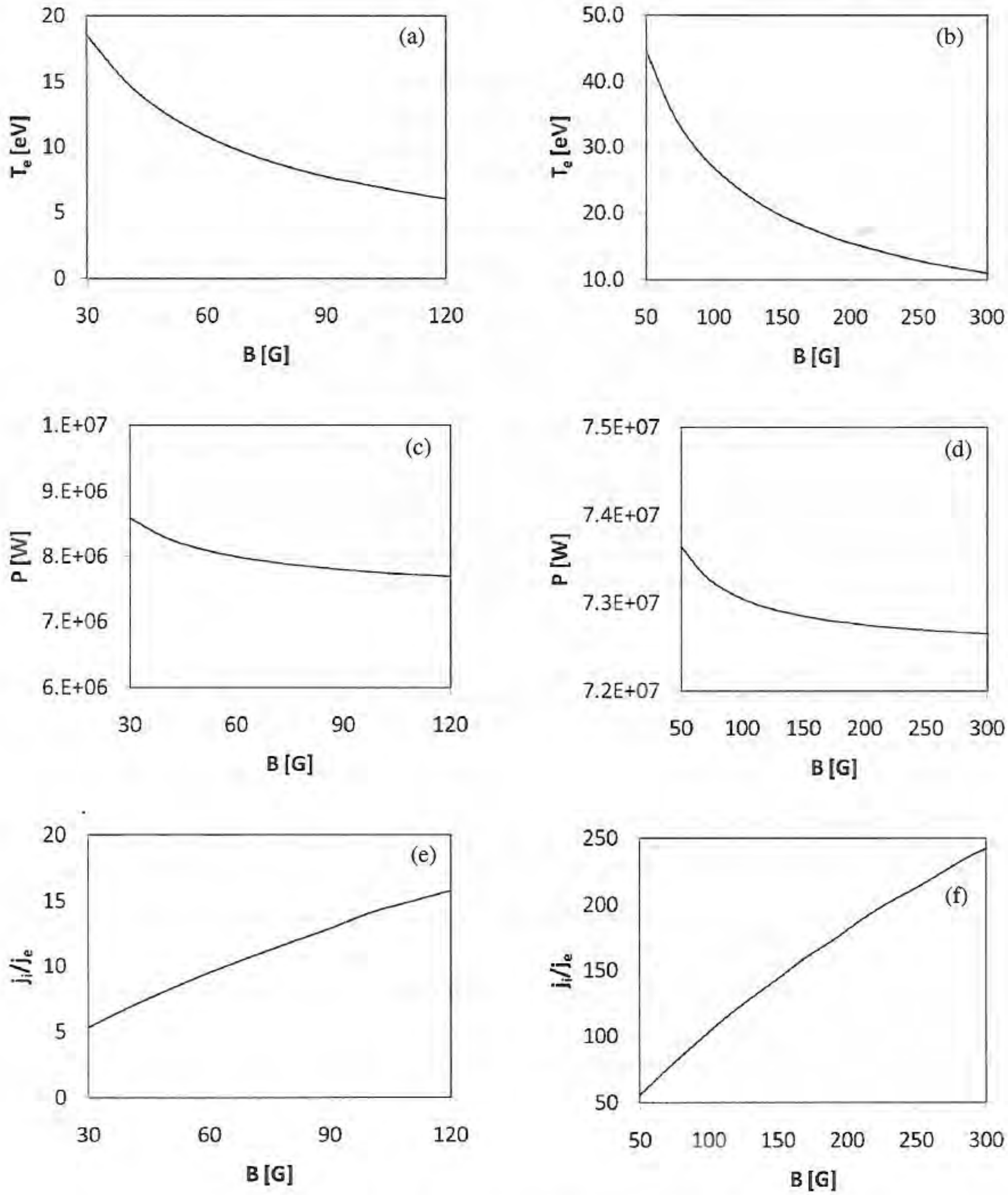

Figure 7. Parameters of the air breathing thruster for H = 80 km: plots (a), (c) and (e) correspond to $\varphi = 3\,kV$ and plots (b), (d) and (f) to $\varphi = 30\,kV$.

the magnetic field (they are magnetized), the electrical power decreases and the ratio of j_i to j_e increases with an increase in B, as illustrated in Figures 6c - 6f and 7c - 7f. Since the electron component of the total current sharply decreases with magnetic field, as shown in Figures 6e, 6f, 7c, 7f, P flattens for large magnetic fields, Figures 6c, 6d, 7c, 7d.

We have also examined the air breathing Hall thruster with the selected L, r_a, and r_b for the satellite orbits of 100 and 110 km. Although we could not select B and φ to satisfy the model assumptions at such high altitude, we believe that changing the geometry of the thruster, for example increasing the length of the thruster, see Eqs. (8) and (12), may allow this.

IV. Concluding Remarks

The idea of using air breathing electrical propulsion for a vehicle flying at orbital speed on the edge of Earth's atmosphere has been examined for a thruster based on the Hall Effect. We have shown that, conceptually, such a thruster can indeed work effectively at the orbits of about 80 – 90 km, producing significant thrust in the range of 20 to 320 N for considered conditions.

It should be pointed out that the power required maintaining such high level of thrust is in the order of MW with thrust-to-power ratio of 4.37 – 13 μN/kW which is comparable with a typical high-efficient electric propulsion device. Recall that while air-breathing concept seems to be reasonably achieved in terms of thrust production it can be realized with high power available only. Thus, analysis of available power generation, power transmission is warranted as a part of the overall air-breathing thruster and satellite analysis.

Note, that this paper has considered relatively simplified model of the plasma-wall interaction inside the thruster chamber. In particular, zero secondary electron emission (SEE) scenario was invoked to describe the sheath formation. Such treatment is limited to the wall materials with a low SEE [10]. Further analysis of the air-breathing concept should involve broader conditions for the SEE thus extending operational parameters of this device.

We believe that by changing the thruster geometry, we could find reasonable conditions allowing increasing the flying orbit of an air breathing satellite up to 100 – 110 km. It should be stressed that in the model we have assumed that the plume is fully ionized corresponding to the maximum possible thrust and power. However, if we soften this condition, assuming that the plume can be only partially ionized, the parameters of the thruster become less "extreme" and may lead to use of the thruster at higher orbital altitude.

References

[1] Diamant K. D., "A 2-Stage Cylindrical Hall Thruster for Air Breathing Electrical Propulsion," *46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit*, 25 -28 July 2010, Nashville, TN, AIAA 2010-6522, 2010.

[2] Hruby V., Pote B., Brogan T., Hohman K., Szabo J., and Rostler P., "Air Breathing Electrically Powered Hall Effect Thruster," Patent, WO 03/098041 A2, 2003.

[3] Moe K. and Moe M. M., "Gas-Surface Interactions and Satellite Drag Coefficients," *Planetary and Space* 53, 2005, pp. 793-801.

[4] Straub H.C., Renault P., Lindsay B. G., Smith K. A., and Stebbings R. F., "Absolute Partial Cross Sections for Electron-Impact Ionization of H_2, N_2, and O_2 from Threshold to 1000 eV," *Physical Review A*, Vol. 54, 1996, pp. 2146.

[5] Itikawa Y., "Cross Sections for Electron Collisions with Nitrogen Molecules," *J. Phys. Chem. Ref. Data*, Vol. 35, 2006, pp. 31-53.

[6] Keidar M. and Beilis I. I, "Electron Transport Phenomena in Plasma Devices with E x B Drift," *IEEE Transaction on Plasma Science*, Vol. 34, 2006, pp. 804-814.

[7] Raizer, Y. P., *Gas Discharge Physics*, New York, 2001.

[8] Bohm D, *The Characteristic of Electrical Discharge in Magnetic Fields*, A Guthrie and R. K. Wekerling, EDs. New York: MacGraw-Hill, 1949, ch. 3, p. 77.

[9] Kennard E H., *Kinetic Theory of Gases*, McGraw-Hill, 1938, p. 149.

[10] M. Keidar, I.D. Boyd and I.I. Beilis, "Plasma flow and plasma-wall transition in Hall thruster channel," *Physics of Plasmas*, 8, No. 12, 2001, pp. 5315-5322.